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The coupling of conduction with forced convection over a flat plate

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Abstract—In this paper, the thermo-fluid-dynamic field resulting from the coupling of laminar forced convection along and conduction inside a heated flat plate is studied by means of two expansions. The first one, describing the field in the leading edge region of the flat plate, is a regular series. The second expansion, which is asymptotic, includes eigensolutions. Moreover, by means of the Padé approximant technique, it is possible to extend the validity of the first expansion over its range of convergence and hence to obtain the description of the entire field.

1. INTRODUCTION

THE REFERENCE work for the conjugated problem of laminar forced convection along a flat plate is due to Luikov *et al.* [1]. In that paper, the coupled conduction problem in a flat plate of finite thickness b and the convection problem for the fluid were considered. The authors solved the problem by means of the generalized Fourier sine transformation and an expansion in series in terms of the Fourier variable. The results of such an analysis were presented in two figures for two examples. This solution cannot be easily used; and Luikov [2] has given an approximate solution of the problem assuming a linear temperature distribution in the plate.

An extension of these results was obtained by Payvar [3] for high Prandtl numbers. An improvement of the Payvar analysis was studied by Karvinen [4], who also presented in ref. [5] an iterative technique for solving the conjugated heat transfer problem in a flat plate in the presence of internal heat sources. Gosse [6] presented an analytic solution which held at high values of the abscissa x. A review of conjugated problems for geometries different from the flat plate was presented by Gori [7].

The purpose of this paper is to describe the entire thermo-fluid-dynamic field by means of two expansions in terms of the coupling parameter: the first one is an initial solution, which holds when the abscissa falls in the range 0–L_{in}, and the second one is an asymptotic solution which holds when the abscissa is greater than L_{as}. The two lengths L_{in} and L_{as} depend on the coupling parameter and the Prandtl number.

By means of the Padé approximant technique, it is possible to match very well these two expansions and to obtain an accurate description of the field.

2. EQUATIONS AND BOUNDARY CONDITIONS

In order to describe the steady two-dimensional forced flow on one side of a flat plate of thickness b, insulated on the edge, and with a temperature T_{b} maintained on the other side (Fig. 1), one must solve the coupled thermal fields in the solid and in the fluid. The coupling conditions require that the temperature and the heat flux be continuous at the interface.

Neglecting the wall axial conduction, the temperature $T_{\text{so}}(x, y)$ in the solid is given by

$$T_{\text{so}}(x, y) = T_{\text{w}} - (T_{\text{b}} - T_{\text{w}})y/b \qquad (1)$$

where $T_{\text{w}} = T(x, 0)$ is the (unknown) temperature at the interface.

The thermo-fluid-dynamic field in the fluid is governed by the boundary layer equations, which in non-dimensional form for a compressible flow may be written as

$$(\rho u)_x + (\rho v)_y = 0 \qquad (2a)$$

$$\rho(uu_x + vu_y) = (\mu u_y)_y \qquad (2b)$$

$$\rho(ut_x + vt_y) = \frac{1}{Pr}(\lambda t_y)_y + (\gamma - 1)M^2 \mu u_y^2 \qquad (2c)$$

where Pr and M are respectively the Prandtl and Mach numbers of the external flow, and $t = T/T_\infty$.

We take as reference lengths the wall thicknesses b

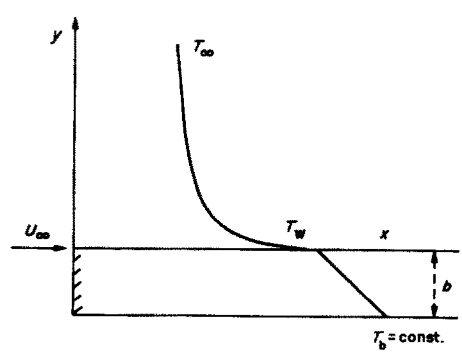

FIG. 1. Thermal model of a flat plate.

NOMENCLATURE

a	$Pr(\gamma-1)M^2/\Delta t_\infty$
A_n, B_n	coefficients of Padé summation, equation (17)
b	plate thickness
C	constant of the first eigensolution ψ
h_i	coefficients of the expansion in MacLaurin series of ϑ, equation (14)
L_{as}	value of the abscissa where the asymptotic solution begins
L_{in}	length of the region in which the initial solution is valid
M	Mach number
m	$p/\xi^{1/2}$
m_1	$1/m$
m_1^+	upper bound of the range of validity of the initial solution
$\tilde{m}_1$	lower bound of the range of validity of the asymptotic solution
$\bar{m}$	value of m where the initial condition for the asymptotic solution is given
Nu_x	local Nusselt number, $-xT_{y,0}/[T_w(x)-T_\infty]$
p	coupling parameter, $\lambda_\infty Re^{1/2}/\lambda_s$
Pr	Prandtl number
Re	Reynolds number, $u_\infty b/v_\infty$
Re_x	Reynolds number, $u_\infty x/v_\infty$
t	dimensionless temperature, T/T_∞
Δt_∞	$(T_b-T_\infty)/T_\infty$
T	temperature
T_b	temperature at the outside surface of the plate
u	fluid axial velocity
v	fluid normal velocity
Δv	$v-V$
V	$\rho v+u\eta_x$
x	dimensionless axial coordinate
y	dimensionless normal coordinate
z	$\eta/\xi^{1/2}$
Z	Blasius solution.

Greek symbols

β_1	first eigenvalue of equation (19)
γ	ratio of specific heats
ϑ	dimensionless temperature, $(T-T_\infty)/(T_b-T_\infty)$
ϑ_i	coefficients of the asymptotic expansion of ϑ, equation (13)
η	$\int_0^y \rho\,dy$
Θ	function defined in equation (22)
λ, λ_s	fluid and solid thermal conductivities
μ	fluid viscosity
v	kinematic viscosity of the fluid
ξ	x
ρ	density of fluid
ψ	eigensolution of equation (23).

Subscripts

w	wall–fluid interface
∞	mainstream flow.

and $bRe^{1/2}$ along the x- and y-directions, respectively, where $Re = u_\infty b/v_\infty$.

The heat flux continuity condition may be written as

$$\lambda Re^{1/2}\, t_y(x,0) = \lambda_s[t_w - t_b] \tag{3}$$

where the thermal conductivities depend in general on the temperature.

The boundary conditions which, together with equation (3), must be associated with the system of equations (2a)–(2c) are

$$u(x,0) = v(x,0) = 0;\quad u(x,\infty) = 1 \tag{4}$$

$$t(0,y) = t(x,\infty) = 1. \tag{5}$$

By assuming that μ and λ vary linearly with temperature and that c_p is a constant, the boundary layer equations reduce to the incompressible form by applying the Stewartson–Dorodnitzin transformation. In this way, equations (2a) and (2b) are independent of equation (2c) and their solution is

$$u = Z';\quad V = (zZ'-Z)/2\xi^{1/2} \tag{6}$$

where $Z(z)$ is the Blasius solution and

$$\xi = x,\quad z = \eta/\xi^{1/2},\quad \eta = \int_0^y \rho\,dy,\quad V = \rho v + u\eta_x. \tag{6'}$$

In terms of the variables ξ and η, the energy equation (2c) may be written as

$$Pr\,(u\vartheta_\xi + V\vartheta_\eta) = \vartheta_{\eta\eta} + au_\eta^2 \tag{7}$$

where $\vartheta = (T-T_\infty)/(T_b-T_\infty)$, $\Delta t_\infty = (T_b-T_\infty)/T_\infty$ and $a = Pr(\gamma-1)M^2/\Delta t_\infty$, and u and V are given by equations (6).

The coupling condition (3) assumes the form

$$p\vartheta_\eta(\xi,0) = \vartheta_w - 1 \tag{8}$$

where $\vartheta_w = \vartheta(\xi,\ 0)$, the coupling parameter $p = \lambda_\infty Re^{1/2}/\lambda_s$, and the thermal boundary conditions (5) become

$$\vartheta(0,\eta) = 0 \tag{9a}$$

$$\vartheta(\xi,\infty) = 0. \tag{9b}$$

3. SOLUTION METHOD

In terms of the variables ξ and z, equations (7) and (8) may be written as

$$(2\xi Z'\vartheta_\xi - Z\vartheta_z)\,Pr = 2\vartheta_{zz} + 2aZ'^2 \qquad (10)$$

$$m\vartheta_z(\xi,0) = \vartheta_w - 1 \qquad (11)$$

where

$$m = p/\xi^{1/2}. \qquad (12)$$

The variable m defined by equation (12) is related to the local Brun number [2] for a flat plate in the following way: $Br_x = m\,Pr^n$ ($n = 1/2$ if $Pr < 0.5$ and $n = 1/3$ if $Pr \geqslant 0.5$). Physically, the local Brun number is proportional to the ratio of the thermal resistances of the wall and boundary layer over the length x.

The boundary condition (11) suggests a change in variables from ξ and z to m and z. If one expands the function ϑ in a MacLaurin series with respect to m ($m \to 0$ corresponds to $x \to \infty$), thus writing

$$\vartheta = \sum_{i=0}^{\infty} m^i \vartheta_i(z) \qquad (13)$$

one finds that this form of the solution is not satisfactory everywhere because m diverges for vanishing ξ. Hence this expansion does not hold at $\xi = 0$, and the initial condition (9a) cannot be satisfied.

Moreover, the linearized problem presents eigenvalues: this circumstance, although it does not permit us to utilize an expansion in terms of m of the form of equation (13), does enable us to solve the problem of the initial conditions. It is necessary to modify this form and to give the boundary condition at $\bar{m}$ ($\bar{m} = p/\xi_0^{1/2}$, where ξ_0 is a suitable positive value of ξ) according to boundary condition (9a).

To obtain this new initial condition, a different expansion (initial expansion) valid for small values of ξ will be considered. The function ϑ is now expanded in a MacLaurin series with respect to $m_1 = 1/m$, thus writing

$$\vartheta = \sum_{i=0}^{\infty} m_1^i h_i(z). \qquad (14)$$

In this way, if one assumes that $h_i(\infty) = 0$, initial condition (9a) is satisfied as well. Moreover, if $\bar{m}$ is a point of convergence of expansion (14), it is possible to obtain in this point the initial condition for a correct expansion in terms of m (asymptotic expansion).

4. EXPANSION FOR SMALL ξ (INITIAL EXPANSION)

By substituting expansion (14) into equations (10) and (11) one finds the following leading-order equation and boundary conditions:

$$h_0'' + Pr\,Zh_0'/2 + aZ''^2 = 0$$

$$h_0'(0) = 0; \quad h_0(\infty) = 0 \qquad (15)$$

and the following for the ith order:

$$2h_i'' - Pr\,(Z'ih_i - Zh_i') = 0$$

$$h_i'(0) = h_{i-1}(0) - \delta_{i1}; \quad h_i(\infty) = 0 \qquad (16)$$

where $\delta_{11} = 1$ and $\delta_{i1} = 0$ for $i > 1$.

Equations (15) and (16) represent a standard boundary-value problem which can be easily solved numerically. It is possible to use the results obtained from series (14) for obtaining, by means of the Padé approximant technique, a new and powerful expansion.

The idea of the Padé summation is to replace a power series $\sum a_n t^n$ by a sequence of rational functions of the form

$$P_M^N(t) = \sum_{n=0}^{N} A_n t^n \Big/ \sum_{n=0}^{M} B_n t^n \qquad (17)$$

where B_0 may be set equal to 1 without loss of generality. The remaining $M+N+1$ coefficients A_n, B_n may be chosen so that the first $M+N+1$ terms in the Taylor series expansion of $P_M^N(t)$ match the first $M+N+1$ terms of the power series $\sum_{n=0}^{\infty} a_n t^n$. The resulting rational function $P_M^N(t)$ is called a Padé approximant and the special sequence for which $M = N$ is called the diagonal sequence.

In this way it is possible to obtain a rapid convergence by using only a few terms of the original Taylor series, but above all the utility of Padé approximants lies in the fact that they also work well when the Taylor series does not converge.

The remarkable improvement with respect to the MacLaurin expansion obtained with Padé approximants will be shown in the following sections.

The solution of the initial problem makes it possible to give the boundary condition at $m = \bar{m}$ for the asymptotic expansion.

5. EXPANSION FOR ξ HIGH (ASYMPTOTIC EXPANSION)

The solution for $m \leqslant \bar{m}$ assumes a form different from that expressed by expansion (13).

In fact, if one substitutes this expansion into equations (10) and (11), one finds the following first-order equation and boundary conditions:

$$2\vartheta_0'' + Pr\,Z\vartheta_0' + 2aZ''^2 = 0$$

$$\vartheta_0(0) = 1; \quad \vartheta_0(\infty) = 0 \qquad (18)$$

and the following for the ith order:

$$2\vartheta_i'' + Pr(Z'i\vartheta_i + Z\vartheta_i') = 0 \qquad (19)$$

$$\vartheta_i(0) = \vartheta_{i-1}'(0); \quad \vartheta_i(\infty) = 0. \qquad (20)$$

Equation (19) presents eigensolutions when associated with the boundary conditions $\vartheta_i(0) = \theta_i(\infty) = 0$.

The first one appears for $1 < \beta_1 = i < 2$, and depends on Pr. For instance, $\beta_1 \simeq 1.60$ for $Pr = 0.70$ and $\beta_1 \simeq 1.51$ for $Pr = 7.02$.

The first two terms in expansion (13) may then be determined by means of equations (18)–(20), and the solution may be written in the form

$$\vartheta = \sum_{i=0}^{1} m^i \vartheta_i(z) + m^{\beta_1} R(m, z) \quad (21)$$

where the function $R(m, z)$ is not analytic with respect to m. R may be represented by a suitable expansion. To estimate the leading term of such an expansion, say $\Theta_1(z)$, we write equation (21) as follows:

$$\vartheta = \sum_{i=0}^{1} m^i \vartheta_i(z) + m^{\beta_1} \Theta_1(z) + o(m^{\beta_1}) \quad (22)$$

where $o(m^{\beta_1})$ denotes terms of order smaller than m^{β_1}.

We now put $\Theta_1(z) = C(\bar{m})\psi(z)$, where $\psi(z)$ is the eigensolution of the equation

$$2\psi'' + Pr\,(Z'\beta_1\psi + Z\psi') = 0 \quad (23)$$

$[\psi(0) = \psi(\infty) = 0]$ that satisfies the initial condition $\psi'(0) = 1$, whereas $C(\bar{m})$ depends on the value of $m = \bar{m}$, where the initial condition for the asymptotic solution is given.

In order to calculate $C(\bar{m})$, we consider the function

$$F(m) = \int_0^{\infty} u\vartheta \, dz.$$

C must be such that the values obtained for such a function at $m = \bar{m}$ by means of the two representations (14) and (22) of ϑ are equal.

6. RESULTS AND DISCUSSION

The initial solution, holding for small values of $m_1 = x^{1/2}/p$ and described by equation (14), has been found with 21 terms of the expansion, using a fourth-order predictor–corrector method. In Table 1, the values of $h_i(0)$, giving the interface temperature ϑ_w for $Pr = 0.7$ (air), for $M \simeq 0$ and $M = 3$, and for $Pr = 7.02$ (water), for $M \simeq 0$, have been listed.

Table 1. Coefficients $h_i(0)$ of expansion (14) (initial solution)

i	$Pr = 0.7$, $M = 0$	$Pr = 0.7$, $M = 3$	$Pr = 7.02$, $M = 0$
0	0	5.8853303	0
1	2.4636984	−12.035983	1.1280761
2	−5.1291153	25.057417	−1.0799054
3	9.5413968	−46.612845	9.2572774E–1
4	−16.313412	79.696306	−7.3030601E–1
5	26.073775	−127.3783	5.3904464E–1
6	−39.399996	192.48139	−3.7639352E–1
7	56.744427	−277.21406	2.5060616E–1
8	−78.362526	382.82461	−1.6004882E–1
9	104.25337	−509.30837	9.8498975E–2
10	−134.12117	655.22015	−5.8632192E–2
11	167.36393	−817.61883	3.3859516E–2
12	−203.09191	992.15765	−1.9017861E–2
13	240.17474	−1173.3142	1.0411339E–2
14	−277.31236	1354.7373	−5.5655834E–3
15	313.12236	−1529.6733	2.9098012E–3
16	−346.23453	1691.4286	−1.4899360E–3
17	375.38318	−1833.8199	7.4809335E–4
18	−399.48843	1951.5720	−3.6872367E–4
19	417.71981	−2040.6281	1.7857745E–4
20	−429.53754	2098.3516	−8.5057695E–5

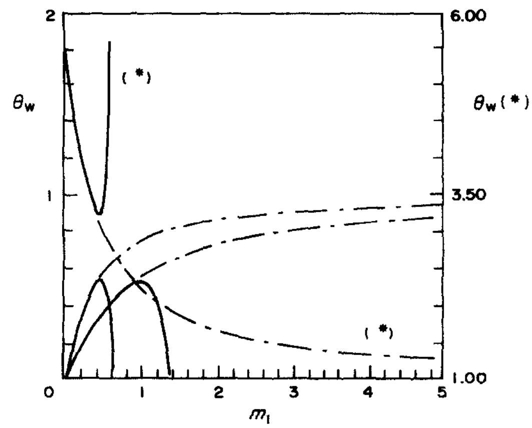

FIG. 2. The interface temperature represented by the initial solution (——) and by Padé summation (—·—) for $Pr = 7.02$ and $M \simeq 0$ (lower curves), $Pr = 0.7$ and $M \simeq 0$ (middle curves), $Pr = 0.7$ and $M = 3$ (upper curves).

By means of these coefficients it was possible to determine the Padé approximants, given by equation (17) with $M = N$, for several values of N. No significant difference was noted between the results obtained for $N = 10$ and those for $N > 10$.

In Fig. 2 the interface temperature ϑ_w represented by the initial solution (14) and the Padé approximants (17) plotted against m_1 is drawn for $Pr = 0.7$ and $M \simeq 0$, for $Pr = 7.02$ and $M \simeq 0$ and for $Pr = 0.7$ and $M = 3$ (* curves). This figure shows that for $m_1 \leqslant m_1^+$ (where $m_1^+ \simeq 0.5$ for $Pr = 0.7$ and $M \simeq 0$, $m_1^+ \simeq 0.9$ for $Pr = 7.02$ and $M \simeq 0$ and $m_1^+ \simeq 0.5$ for $Pr = 0.7$ and $M = 3$), the two representations give very similar results, while for $m_1 > m_1^+$, the results are completely different. If we denote by L_{in} the length of the strip in which the initial solution holds, $L_{in} = (m_1^+ p)^2 b$. We shall show later that, by comparison with the asymptotic values, the Padé curve converges to the exact one.

Let us consider now the asymptotic solution (22) represented in terms of the variable $m = p/x^{1/2}$. The values of $\vartheta_0'(0)$ and $\vartheta_1'(0)$ for $Pr = 0.7$ ($M \simeq 0$ and $M = 3$) and for $Pr = 7.02$ ($M \simeq 0$) are given in Table 2.

These coefficients enable us to obtain the asymptotic solution, neglecting terms of order m^{β_1}; an improvement in these results may be obtained by

Table 2. Coefficients $\vartheta_i'(0)$ of asymptotic expansion (22)

i	$Pr = 0.7$, $M \simeq 0$	$Pr = 0.7, M = 3$, $\Delta t_\infty = 0.2556$	$Pr = 7.02$, $M \simeq 0$
0	−0.292680	1.429840	−0.646542
1	0	0	0

Table 3. Comparison of the Padé summation, the asymptotic solution with two terms and the asymptotic solution with three terms

m_l	Padé summation	Asymptotic solution two terms	three terms
0.5	−0.2088	−0.2927	−0.1653
1.0	−0.2530	−0.2927	−0.2507
1.5	−0.2692	−0.2927	−0.2707
2.0	−0.2769	−0.2927	−0.2788
3.0	−0.2839	−0.2927	−0.2854
4.0	−0.2870	−0.2927	−0.2881
6.0	−0.2897	−0.2927	−0.2903
8.0	−0.2909	−0.2927	−0.2912
10.0	−0.2915	−0.2927	−0.2916

adding the first eigensolution, as estimated at the end of Section 5.

Table 3 compares the values of $\vartheta_z(m_l, 0)$ obtained by means of the Padé summation (17), the asymptotic solution (22), neglecting terms of order m^{β_l}, and the asymptotic solution, neglecting terms smaller than m^{β_l} (constant C has been evaluated at $\bar{m} = 0.33$) for $Pr = 0.7$ and $M \simeq 0$.

In Fig. 3, the interface temperature ϑ_w represented by the Padé summation and by the asymptotic solution with two terms is plotted against m_l for $Pr = 0.7$ and $M \simeq 0$, for $Pr = 7.02$ and $M \simeq 0$ and for $Pr = 0.7$ and $M = 3$. Table 3 and Fig. 3 show that for $m_l \geqslant 8$ the values given by the Padé representation practically coincide with those given by the asymptotic solution. Therefore, while the MacLaurin initial expansion holds for $0 \leqslant m_l \leqslant m_l^\dagger$ the Padé representation holds in the entire field.

Figure 4 compares the interface temperature ϑ_w obtained from the present solution (solid curves) with that of ref. [1]. These solutions refer to different situations: the present one may be considered exact, starting from assumption (1), while the Luikov solution, obtained without assumption (1), is based on an

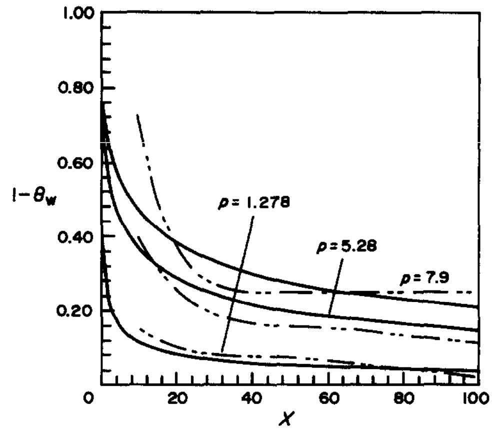

FIG. 4. Comparison between the present results (——) and the Luikov [1] results (– – – –).

expansion in series the accuracy of which is not discussed.

In Fig. 5, the influence of the Mach number on the interface temperature and on the Nusselt number is shown, subject to the hypothesis of no interference between the shock wave and the boundary layer.

The local Nusselt number Nu_x is defined as $Nu_x = -xT_{y,0}/[T_w(x) - T_\infty]$. In Fig. 6, the value of Nu_x obtained from the present solution (solid curves) is compared, for several Prandtl numbers, with that obtained from the first-order asymptotic solution of ref. [6] (dashed curves), in which the expression presented for $0.6 < Pr < 15$ is

$$\frac{Nu_x}{Re_x^{1/2}} = \frac{0.332 Pr^{1/3} m_l}{m_l - 0.332 Pr^{1/3}}.$$

This figure shows that for $m_l > \tilde{m}_l$, the solution of ref. [6] is accurate (for $Pr = 0.7$, $\tilde{m}_l = 4$; for $Pr = 2$, $\tilde{m}_l = 6$; for $Pr = 7.02$, $\tilde{m}_l = 8$), and for $m_l \to \infty$, the curves of Nu_x tend to values which

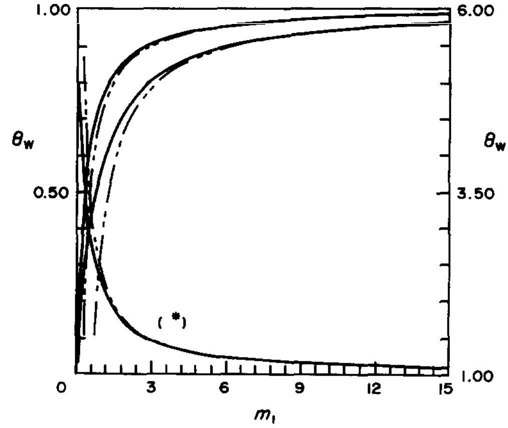

FIG. 3. The interface temperature in the asymptotic (– – – –) and Padé representation (——) (for $Pr = 0.7$ and $M \simeq 0$ (upper curves), $Pr = 7.02$ and $M \simeq 0$ (middle curves), $Pr = 0.7$ and $M = 3$ (lower curves).

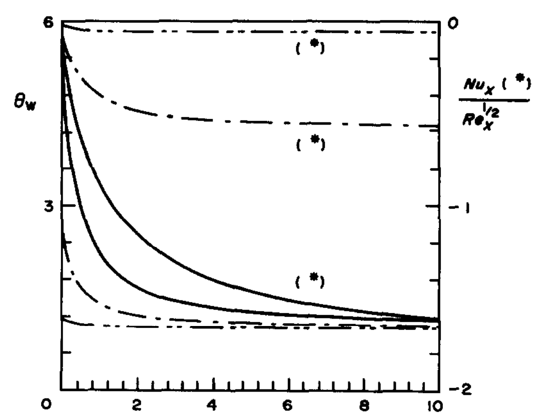

FIG. 5. The influence on the interface temperature and on the Nusselt number of M for $Pr = 0.7$ and $\Delta t_\infty = 0.2556$: ——, $M = 3$; —·—, $M = 2$; – – – –, $M = 1.34$.

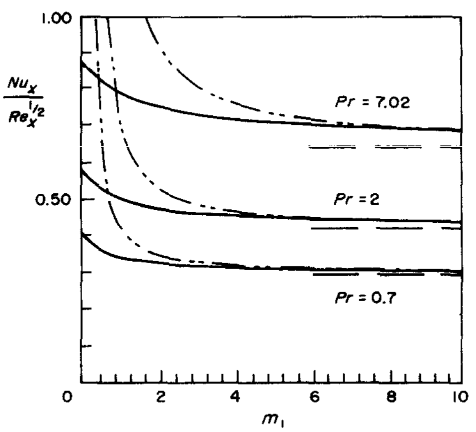

FIG. 6. Comparison between the present results (——), the Gosse [6] results (— — —) and the approximate expression (— — —).

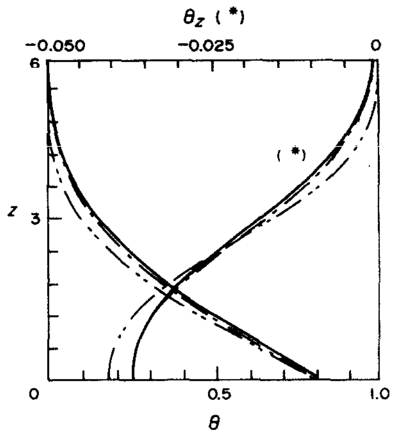

FIG. 8. Profiles of ϑ and ϑ_z vs z for $m_1 = 1.41$ obtained from the Padé summation (——), the two-term asymptotic expansion (— — —) and the three-term asymptotic expansion (—·—) ($Pr = 0.7$ and $M \simeq 0$).

are well described by the approximate expression $Nu_x = 0.332 Re_x^{1/2}\, Pr^{1/3}$ holding in the isothermal case (dashed lines). Thus the asymptotic solution is valid for values of the abscissa greater than L_{as}, where $L_{as} = (\tilde{m}_1 p)^2 b$.

Padé's approximant technique may also be used for determining the temperature and velocity profiles.

In fact, if we put

$$\sum_{i=0}^{M+N} m_1^i h_i(z) = \sum_{i=0}^{N} A_i m_1^i \Big/ \sum_{i=0}^{M} B_i m_1^i$$

then the Padé coefficients A_i, B_i will depend on z.

By computing these coefficients for several values of z, it is possible to draw the temperature profiles for each value of m_1.

In Figs. 7–9, the Padé representation of the profiles of ϑ and θ_z for $Pr = 0.7$ and $M \simeq 0$ are plotted against z for $m_1 = 1$, 1.41 and 4, together with those obtained from the two-term asymptotic expansion, equation (13), and from equation (22) by calculating $C(\tilde{m})$ at $\tilde{m} = 0.33$.

These figures show that at small values of m_1, the asymptotic solution is not accurate when represented by means of two terms of equation (13), but that it improves appreciably when the first eigensolution is added. Moreover, the profiles obtained by the Padé representation are very close to the correct asymptotic ones (they practically coincide for $m_1 \geqslant 4$), and hence the Padé representation is valid in the entire field.

In order to determine the velocity profiles, the correspondence between the variables (m_1, z) and (x, y) must be obtained.

From the definitions of $m_1 = 1/m$ and z given by equations (12) and (6′), respectively, one has

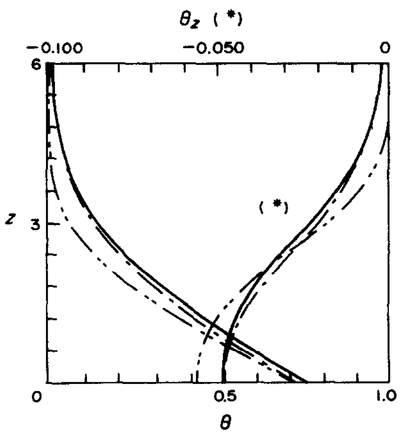

FIG. 7. Profiles of ϑ and ϑ_z vs z for $m_1 = 1$ obtained from the Padé summation (——), the two-term asymptotic expansion (— — —) and the three-term asymptotic expansion (—·—) ($Pr = 0.7$ and $M \simeq 0$).

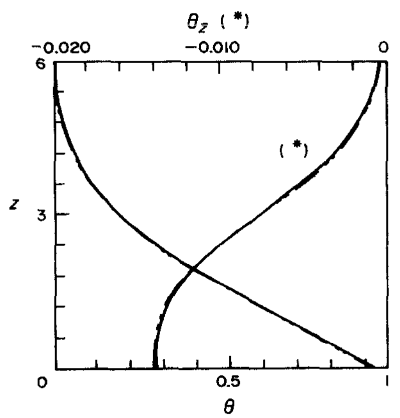

FIG. 9. Profiles of ϑ and ϑ_z vs z for $m_1 = 4$ obtained from the Padé summation (——), the two-term asymptotic expansion (— — —) and the three-term asymptotic expansion (—·—) ($Pr = 0.7$ and $M \simeq 0$).

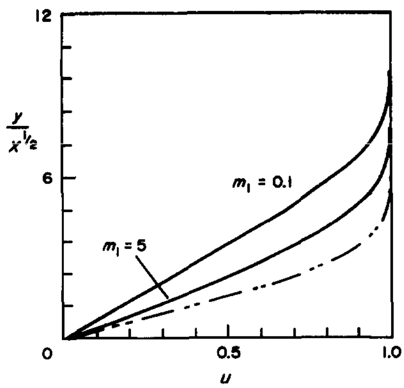

FIG. 10. Velocity profiles vs $y/x^{1/2}$ for $Pr = 0.7$, $M = 3$ and $\Delta t_\infty = 0.2556$ compared with that obtained for $Pr = 0.7$ and $M \simeq 0$ (dashed curve).

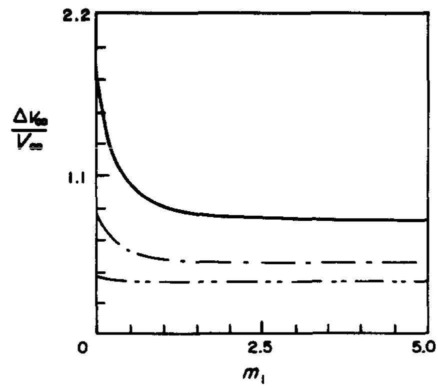

FIG. 11. The influence on $\Delta v_\infty/V_\infty$ of M for $Pr = 0.7$ and $\Delta t_\infty = 0.2556$: ——, $M = 3$; —·—, $M = 2$; – – – –, $M = 1.34$.

$$y/x^{1/2} = z+\Delta t_\infty \sum_i m_1^i \int_0^z h_1(z)\,dz.$$

Equations (6) then give the velocity profiles. In Fig. 10, the u component for $m_1 = 0.1$ and 5, together with the incompressible one (dashed curve), is plotted against $y/x^{1/2}$

The transverse velocity may be obtained from equation (6′), where η_x is given by

$$\eta_x(m_1, z) =$$

$$-\xi^{-1/2}\frac{\Delta t_\infty}{2}\frac{\Sigma(i+1)m_1^i \int_0^z h_i(z)\,dz - z\Sigma m_1^i h_i(z)}{1+\Delta t_\infty \Sigma m_1^i h_i(z)}.$$

In Fig. 11, $\Delta v/V = (v-V)/V$ for $y \to \infty$, given by

$$\Delta V_\infty/V_\infty = \Delta t_\infty \sum_i (i+1)m_1^i \int_0^\infty h_i(z)\,dz/(zZ'-Z)_\infty$$

is plotted for $M = 1.34$, 2 and 3 against m_1.

7. CONCLUDING REMARKS

In this paper an accurate solution of the coupled forced convection–conduction problem for a flat plate has been given.

This result was obtained starting from two expansions for the temperature, holding for small and high values of the abscissa, and after using the Padé approximant technique.

The initial solution is valid when the abscissa falls in the range 0–L_{in}, where $L_{in} = (m_1^+ p)^2 b$, and the asymptotic one is valid when the abscissa is greater than L_{as}, where $L_{as} = (\tilde{m}_1 p)^2 b$. For $Pr = 0.7$, $m_1^+ = 0.5$ and $\tilde{m}_1 = 4$, and for $Pr = 7.02$, $m_1^+ = 0.9$ and $\tilde{m}_1 = 8$. The values of interest in practical applications of the coupling parameter p are in the range 10^{-2}–10^2.

A comparison of the Padé representation with the MacLaurin initial solution, holding for small values of the abscissa, and with the asymptotic solution, holding for high values of the abscissa, showed that this representation is valid in the entire field.

Once the temperature distribution was known it was possible to determine the velocity profiles by means of the Stewartson–Dorodnitzin transformation.

The present solution was compared with those in the literature, and the influence of the Mach number on the interface temperature, the Nusselt number and the velocity field was demonstrated.

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COUPLAGE DE LA CONDUCTION AVEC LA CONVECTION FORCEE SUR UNE PLAQUE PLANE

Résumé—Le champ thermo-fluido-dynamique qui résulte du couplage de la convection laminaire forcée le long d'une plaque plane chauffée et de la conduction à l'intérieur est étudié à l'aide de deux développements. Le premier est un développement en série régulière qui décrit le champ dans la région du bord d'attaque de la plaque. Le second qui est asymptotique inclut les fonctions propres. Au moyen de la technique de Padé il est possible d'étendre la validité du premier développement sur son domaine de convergence et d'obtenir la description du champ complet.

KOPPLUNG VON WÄRMELEITUNG MIT ERZWUNGENER KONVEKTION ÜBER EINER EBENEN PLATTE

Zusammenfassung—In dieser Arbeit wird das thermo-fluid-dynamische Feld, das aus der Kopplung von laminerer erzwungener Konvektion an und von Wärmeleitung in einer beheizten ebenen Platte resultiert, mit Hilfe zweier Entwicklungen untersucht. Die erste, die das Feld an der Anströmkante der ebenen Platte beschreibt, ist eine regelmäßige Reihe. Die zweite Entwicklung beinhaltet Eigenlösungen und ist asymptotisch. Weiterhin ist es mit Hilfe der Näherungsmethode nach Padé möglich, die Gültigkeit der ersten Entwicklung über ihren Konvergenzbereich hinaus zu erweitern und so die Beschreibung des gesamten Feldes zu erhalten.

СОПРЯЖЕНИЕ ТЕПЛОПРОВОДНОСТИ С ВЫНУЖДЕННОЙ КОНВЕКЦИЕЙ НАД ПЛОСКОЙ ПЛАСТИНОЙ

Аннотация—Методом двух разложений исследуется термогидродинамическое поле, возникающее при сопряжении ламинарной вынужденной конвекции вдоль нагретой плоской пластины с теплопроводностью внутри пластины. Первое разложение, описывающее поле в области передней кромки пластины, представляет собой обычный ряд. Второе, являющееся асимптотическим, включает собственные решения. Кроме того, с помощью техники Паде-аппроксимации удалось распространить применимость первого разложения за пределы области его сходимости и таким образом получить описание всего поля.